Solutions to wild four-digit quaternaries

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Game	Period	Q-Size
0.0122	7	20
0.0123	7	20
0.1023	7	20
0.1032	7	20
0.1033	7	20
0.1231	5	20
0.1232	6	46
0.1233	6	46
0.1321	5	20
0.1323	6	46
0.1331	5	20

Game	Period	Q-Size
0.2012	5	20
0.2112	5	20
0.3101	2	14
0.3102		
0.3103	5	20
0.3112	5	20
0.3122		
0.3123		
0.3131	2	12
0.3312		

$$|\mathcal{Q}_{0.3102}(11)| = 74$$
$$|\mathcal{Q}_{0.3122}(6)| = 52$$
$$|\mathcal{Q}_{0.3123}(11)| = 328$$
$$|\mathcal{Q}_{0.3312}(13)| = 264$$

Figure 1: Summary of Wild Four-Digit Quaternaries.

	0	1	2	3	4	5	6
0	1	a	1	1	b	b	a
7	a	d^2	1	1	c	d	a
14	a	d^2	1	1	c	d	a
21	a	d^2	1		$\cdots$		

0.1023

	0	1	2	3	4	5	6
0	1	a	1	a	1	b	b
7	b	a	d^2	d^2	1	c	ad^2
14	d	a	d^2	d^2	1	c	ad^2
21	d	a	d^2	d^2	1	$\cdots$	

0.1032

	0	1	2	3	4	5	6
0	1	a	1	a	b	b	b
7	1	d^2	d^2	a	d	ad^2	c
14	1	d^2	d^2	a	d	ad^2	c
21	1	d^2	d^2	a		$\cdots$	

0.1033 (0.0122, 0.0123)

	0	1	2	3	4
0	1	a	1	b	b
5	a	d^2	1	c	d
10	a	d^2	1	c	d
15	a	d^2	1	c	$\cdots$

0.1231 (0.1321)

	0	1	2	3	4
0	1	a	a	b	b
5	1	d^2	a	d	c
10	1	d^2	a	d	c
15	1	d^2	a	d	$\cdots$

0.1331

	0	1	2	3	4
0	1	a	b	d^2	d
5	1	a	c	d^2	d
10	1	a	c	d^2	d
15	1	a		$\cdots$	

0.3103 (0.2012, 0.2112)

	0	1	2	3	4
0	1	a	b	a	1
5	b	d^2	c	a	1
10	d	d^2	c	a	1
15	d	d^2	c	a	1

0.3112

$$\mathcal{Q}_{20} = \{a, b, c, d : a^2 = c^2 = 1,\ b^3 = b^2c,\ d^3 = ad^2,\ bcd = b^2d = d\}$$

$$\mathscr{P} = \{a, b^2, ac, bd, d^2\}$$

Figure 2: Pretending functions for quaternaries with misère quotient group $\mathcal{Q}_{20}$ (first cousins in parentheses).

	0	1	2	3
4	1	a	b	c^2
8	c	c^2	ac^2	c^2
12	ac^2	c^2	ac^2	c^2
16	ac^2		$\cdots$	

0.3101

	0	1	2	3
4	1	a	b	a
8	b	b^2	c	b^2
12	ab^2	b^2	ab^2	b^2
16	ab^2	b^2	ab^2	$\cdots$

0.3131

$$\mathcal{Q}_{0.3101} = \{a, b, c : a^2 = 1,\ b^3 = b,\ c^3 = ac^2,\ b^2c = c\},\ \mathscr{P} = \{a, b^2, bc, c^2\}$$

$$\mathcal{Q}_{0.3131} = \{a, b, c : a^2 = c^2 = 1,\ b^3 = b^2c\},\ \mathscr{P} = \{a, b^2, ac\}$$

Figure 3: Pretending functions for quaternaries with period 2.

	0	1	2	3	4	5
0	1	a	1	b	b	ab
6	a	b^2	1	c	b	d
12	a	b^2	1	c	b	d
18	a	b^2	1	c	$\cdots$	

0.1232, 0.1233

	0	1	2	3	4	5
0	1	a	a	1	b	b
6	ab	a	b^2	1	c	b
12	d	a	b^2	1	c	b
18	d	a	b^2	1	c	$\cdots$

0.1323

$$\mathcal{Q}_{46} = \{a, b, c, d : a^2 = 1,\ b^3 = b^2c,\ bc^3 = bc,\ bcd = b^2d,\ b^3d^2 = cd^2 = bd^2,\ b^2d^3 = d^3, d^5 = ad^4\}$$

$$\mathscr{P} = \{a, b^2, ac, bd, cd, ac^2, ad^2, ac^3, b^2d^2, d^4, ab^3d, abd^3\}$$

Figure 4: Pretending functions for quaternaries with misère quotient group $\mathcal{Q}_{46}$.

	0	1	2	3	4	5	6	7	8	9
0	1	a	1	1	a	b	b	b	b	c
10	c	a	ab^2	ab^2	d	b	b	b	b	c^3
20	e	ab^2	ab^2	ab^2	d	b	b	b	b	c^3
30	e	ab^2	ab^2	ab^2	d			$\cdots$		

$$\mathcal{Q} = \{a, b, c, d, e : a^2 = 1,\ b^3 = b,\ c^4 = c^2,\ d^2 = b^2,\ e^2 = c^2,\ b^2c = c,\ bd = ab,\ cd = ac,\ c^2e = c^3,\ de = ab^2e\} \text{ (30 elements)}$$

$$\mathscr{P} = \{a, b^2, c^2, ae, abce\}$$

0.144

Figure 5: Pretending function for 0.144.